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Intertemporal Incentive, Career Concern, and Promotion

Tsung-Yuan Wang and Chia-Ming Liu

Department of International Business, Chang Jung Christian University, Tainan, Taiwan

Abstract

We analyze how a principal should set his promotion rule when the agents have career concerns and compete with each other to get promoted. When the principal uses promotion as an incentive scheme to increase the agents' effort, the rule may or may not favor the agent with higher expected ability. However, when the principal uses promotion to pick the best agent, the rule always gives higher promotion probability to the agent with higher expected ability.

Keywords: promotion, career concern, incentive

Corresponding Author:
Chia-Ming Liu
cm2815@mail.cjcu.edu.tw

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1. Introduction

Although the principal-agent literature prescribe the use of contracts linking agents' performance and reward to provide work incentive, not only the explicit incentives are rarely seen in governments, where a worker's pay is based on his seniority and job level, but Medoff and Abraham (1980) find no evidence that private firms adopt the pay-for-performance system. Baker, Jensen and Murphy (1988) argue that the pay-for-performance system is not popular because it is so powerful that it creates adverse side effects: workers putting effort only in the activities that count toward their performance. (Holmstrom and Milgrom (1991) provide a theory to explain this effect), lobbying or bribing for better scores (the influence behavior. (See Milgrom and Roberts (1992))), and hoarding effort to have an easier target next year (the ratchet effect. (See Freixax, Guesnerie and Tirole (1985))). Furthermore, the system faces the difficulties in specifying objective performance measurement, and has to deal with the mistrust when it relies instead on subjective measurement.

Besides the explicit incentive, in dynamic settings the principal can resort to implicit incentives, including promotion-based incentives and agents' career concern – working in order to impress the job market. When an organization does not use explicit incentives and use promotions instead, Bakeret al (1988) argue that promotions are used for two purposes: first to provide incentive for the agents who value the higher positions to work harder; second, to screen employees so that agents are assigned to the jobs that fit them best.

However, considering the promotion process as a tournament, they claim that the two roles of promotions are not compatible. For those who have no chance of winning the positions, or those who have sure chance of winning, promotion cannot provide incentive. When employees are alike, the tournament for promotion can provide most incentive, but then the matching is not important. When people are different, the tournament offers little signal about who is the best for the higher-level job. Worse yet, when the requirements for the higher-level job differ much from those in the tournament stage, the winner in the tournament is not likely to be the right person; namely, the Peter Principle occurs.

Since promotions occur in dynamic settings, the agents have career concern as well when they choose their effort facing promotion-based incentives. Therefore promotions might perform better than what (Baker et al., 1988) picture. Furthermore a principal can change the rules of promotions to fit his goal. Our paper aims to analyze the incentives of agents when both the tournament effect and the career concern effect are present, and the principal's optimal promotion rule.

So far, it seems that the papers discussing tournament leave career concern untouched, and those analyzing career concern have no tournaments. Lazear and Rosen (1981) model promotions as one-period tournaments, and argue that when it is costly to monitor agents' effort and outputs, risk-averse agents prefer the reward system based on relative performance (rank-based) to one based on absolute performance. Green and Stokey (1983) analyze how the information structure affects the advantage of tournaments. A tournament performs better when the common shock is important, but worse when the shock is unimportant.

Malcomson (1984) traces the advantage of rank-based reward system to the principal's commitment to giving rewards, which in the output-based reward system he can forfeit by claiming that none fulfills the standard. The reward in a tournament can be purely monetary, not necessarily promotion. Fairburn and Malcomson (1994) show that if the reward is monetary, the workers might put effort in bribery or other influence activity. If the reward is promotion, and the higher-level job is important to the organization, then the organization will be less affected by influence activity since the personal favor the principal gets is small, compared with the organization's future. Fairburn and Malcomson (2001) show that when the agents are risk averse, there can be too little or too much promotion. Fama (1980) first discusses career concerns. He conjectures that the labor market provides enough discipline that contracts are not needed. Holmstrom (1999) shows that the incentive from career concerns is too much for an agent in the early stage of his career, and too little for one near retirement. Hence it is necessary to have contract. In different setups, Holmstrom and Costa (1986) and Gibbons and Murphy (1992) discuss optimal incentive contracts when there is career concern.

Costa (1988) introduces the signaling role of job assignment and promotion. The job market observes what a worker's job is and whether he is promoted, but not his output, which only his current employer observes. So the worker's internal reputation differs with his external reputation. The model shows that there is a misassignment of the worker to jobs and too little promotion. Dewatripont, Jewitt and Tirole (1999) analyze the connection between workers' career concern and job assignment in governments. Meyer and Vickers (1997) consider the interactions between the explicit incentives, which are contracts linking performance to pay, and the implicit incentives, including career concern and the ratchet effect.

A principal can manipulate the rules of promotion to fit his goals. Meyer (1991) shows that when effort do not affect output in a tournament of several rounds, it is optimal to favor the leader of early rounds by giving him advantage in the later rounds. However, Lazear and Rosen (1981) think it is necessary to handicap the leader in the tournament so that both the leader and the followers have incentive to keep on competing. We will explore the optimal bias in the promotion when there are both career concern and tournament effects.

We proceed as follows. In section 2 we lay out our model. In section 3 we derive the workers' equilibrium effort given a promotion rule. Then we do the comparative statics in section 4. In section 5 we discuss the optimal promotion rules under two different goals. Section 6 is the conclusion.

2. The Model

Consider a firm (the principal) who hires two workers, $i = x, z$. There are two periods, $t = 1, 2$. In period 1, the two workers are assigned the same job L. At the start of period 2, after observing the two workers' period-1 outputs, the firm promotes one worker to job H and assigns the other the same job L. In both periods, worker i's output in job L, y^L_{it}, is the sum of his ability θ_i, his effort a^L_{it}, and disturbance ε^L_{it},

$$y^L_{it} = \theta_i + a^L_{it} + \varepsilon^L_{it} \tag{2.1}$$

Output in job H, y_i^H, is affected more by workers' ability. Given the worker's ability, his effort a_i^H, and disturbance ε_i^H, the output is

$$y_{it}^H = \theta_i + a_{it}^H + \varepsilon_{it}^H \tag{2.2}$$

where $\gamma > 1$, Hence, other things equal, the firm will promote the worker with higher ability. The firm and the workers are risk neutral. The cost of effort to worker i in job L is $\beta(a_{it}^L)^2/2$, and is $\beta(a_{it}^H)^2/2$ in job H, $\beta > 0$.

Before production begins, none of the firm, the labor market, and the workers know either worker's ability, and they believe that θ_i is normally distributed with mean $\overline{\theta}_i$ and variance σ_i^2, $i = x, z$ and θ_x is independent of θ_z. Note that the two workers' expected ability can be different, which might be due to different schooling or training of workers before being hired by the firm. The disturbance terms are normally distributed with mean zero and variance $(\varphi_i^j)^2$, $j = 1, 2$ and are independent of θ_i. Let $\epsilon = (\varepsilon_x^L, \varepsilon_z^L)$), then the covariance matrix of the two disturbance terms is

$$E\left(\epsilon'\epsilon\right) = \begin{bmatrix} (\varphi_x^L)^2 & \rho\varphi_x^L\varphi_z^L \\ \rho\varphi_x^L\varphi_z^L & (\varphi_z^L)^2 \end{bmatrix}$$

where ρ is the coefficient of correlation. If $\rho = 0$, then the two workers' outputs in period 1 are independent variables. Let$\Delta y = y_{x1}^L - y_{z1}^L$, $\Delta\theta = \theta_x - \theta_z$,$\Delta\varepsilon = \varepsilon_{x1} - \varepsilon_{z1,}$ and $\Delta a = a_{x1}^L - a_{z1}^L$, then

$$\Delta y = \Delta\theta + \Delta a + \Delta\varepsilon \tag{2.3}$$

Let $V(\Delta\theta)$, $V(\Delta\epsilon)$, and ϑ be the variance of $\Delta\theta$, $\Delta\epsilon$ and Δy respectively, then $V(\Delta\theta) = \sigma_x^2 + \sigma_z^2$; $V(\Delta\epsilon) = (\varphi_x^L)^2 + (\varphi_z^L)^2 - 2\rho\varphi_x^L\varphi_z^L$, and $\vartheta = V(\Delta\theta) + V(\Delta\epsilon)$. The timing of the two-period game is as follows. We depict the promotion rule by a number c. In period 1, the firm announces before production starts that worker x will be promoted if $\Delta y > c$, z be promoted if $\Delta y < c$, and each has equal probability to be promoted if $\Delta y = c$. If $c > 0$ $(c < 0)$, then the rule favors worker $z(x)$. Worker i receives a wage w_{i1}, and exerts effort $a_{i1.}^L$ The firm cannot observe the effort of workers, but their outputs are observable by the firm and the labor market. The outputs are not verifiable and therefore w_{i1} cannot be contingent on outputs.

In period 2, the promoted worker gets a wage W_i and spend effort a_i^H; the other worker gets w_{k2}, and puts in effort a_{k2}^L. Then the outputs are realized. Since the labor market has the same information about the workers' ability, the workers cannot get a higher wage from other firms and so they stay with the firm if the wage offers are the same. Assume further that the firm will not hire someone from outside to fill in the job H. The price of outputs is 1, and there is no discounting between the two periods. We solve the game by backward induction. First we derive workers' effort in the two periods as functions of c, the promotion rule, and then solve the firm's optimal c.

3. The Efforts of Workers

We start with period 2. After observing period-1 outputs y_{x1}^L and y_{z1}^L, if the firm (and the labor market) presumes that $\overline{a}_{x1}^L$ and $\overline{a}_{z1}^L$ were the efforts of workers, then he can subtract from y_{x1}^L and y_{z1}^L the workers' effort to update his belief about their ability. Denote $\Delta a = \overline{a}_{x1}^L - \overline{a}_{z1}^L$ the expected effort difference, and $\Delta\overline{\theta} = \overline{\theta}_x - \overline{\theta}_z$ the difference of the means of the workers' ability, then the firm expects the distribution of Δy to be normally distributed as $N(\Delta\overline{a} + \Delta\overline{\theta}, \vartheta)$. Let the variance of y_{i1}^L be τ_i, then $\tau_i = \sigma_i^2 + (\varphi_i^L)^2$. Given the period 1 outputs, the firm and both workers update their beliefs about both the ability of workers as:

$$E\left(\theta_i \mid y_{x1}^L, y_{z1}^L, \overline{a}_{x1}^L, \overline{a}_{z1}^L\right) = \overline{\theta}_i + A_i\left(y_{i1}^L - \overline{\theta}_i - \overline{a}_{i1}^L\right) + B_i\left(y_{i1}^L - \overline{\theta}_k - \overline{a}_{k1}^L\right), \tag{3.1}$$

$$i = x, z; k \neq i$$

Where

$$A_i = \frac{\sigma_i^2\tau_k}{\tau_i\tau_k - \rho^2(\varphi_i^L)^2(\varphi_k^L)^2} = \frac{\sigma_i^2\tau_k}{\sigma_i^2\tau_k + \sigma_k^2(\varphi_i^L)^2 + (1-\rho^2)(\varphi_i^L)^2(\varphi_k^L)^2} < 1 \tag{3.2}$$

$$B_i = \frac{-\rho\varphi_i^L\varphi_k^L\sigma_i^2}{\tau_i\tau_k - \rho^2(\varphi_i^L)^2(\varphi_k^L)^2} \tag{3.3}$$

Since period 2 is the last period, neither worker exerts any effort. Therefore the wages equal the expected outputs conditional on no effort. The promoted worker i gets

$$W_i = E\left(y_i^H \mid y_{x1}^L, y_{z1}^L, \overline{a}_{x1}^L, \overline{a}_{z1}^L\right) = \gamma E\left(\theta_i \mid y_{x1}^L, y_{z1}^L, \overline{a}_{x1}^L, \overline{a}_{z1}^L\right) \tag{3.4}$$

The unpromoted worker k gets

$$w_{k2} = E\left(y_{k2}^L \mid y_{x1}^L, y_{z1}^L, \overline{a}_{x1}^L, \overline{a}_{z1}^L\right) = E\left(\theta_k \mid y_{x1}^L, y_{z1}^L, \overline{a}_{x1}^L, \overline{a}_{z1}^L\right) \tag{3.5}$$

Since the workers exert effort only in period 1, we drop the superscript L from the effort terms a_{i1}^{L}, and from the variance terms φ_{i1}^{L}. In period 1, the two workers choose effort simultaneously. Denote by b_z worker x's conjecture about z's effort, and b_x worker z's conjecture about x's effort. From worker x's point of view, Δy is distributed as $N(a_{x1} - b_z + \Delta\theta, \vartheta)$; and from z's view, the distribution is $N(b_x - a_{z1} + \Delta\theta, \vartheta)$. Let $g(\bullet)$ be the density function of Δy, then before worker x chooses his effort, his total expected utility is

$$U_x = \left(a_{x1}, b_z\right) = w_{x1} + \int_c^{\infty} W_x g\left(\Delta y \mid a_{x1}, b_z\right) d\Delta y + \int_{-\infty}^{c} w_{x2} g\left(\Delta y \mid a_{x1}, b_z\right) d\Delta y - \frac{\beta a_{x1}^2}{2} \quad (3.6)$$

Where the second term is his expected income from being promoted, which occurs if $\Delta y > c$, and the third term the expected income if not. Similarly worker z's expected utility is

$$U_z = \left(a_{z1}, b_x\right) = w_{z1} + \int_{-\infty}^{c} W_z g\left(\Delta y \mid a_{z1}, b_x\right) d\Delta y + \int_c^{\infty} w_{z2} g\left(\Delta y \mid a_{z1}, b_x\right) d\Delta y - \frac{\beta a_{z1}^2}{2} \quad (3.7)$$

To simplify the above equations, we normalize the distribution $g(\bullet)$ to be standard normal by the transformation

$$\alpha_x\left(\Delta y, a_{x1,} b_z\right) = \frac{\Delta y - E[\Delta y | a_{x1}, b_z]}{\sqrt{\vartheta}}$$

$$\alpha_z\left(\Delta y, a_{z1,} b_x\right) = \frac{\Delta y - E[\Delta y | a_{z1}, b_x]}{\sqrt{\vartheta}}$$

then we have

$$\begin{aligned} U_x\left(a_{x1}, b_z\right) &= w_{x1} + \gamma\left[\overline{\theta}_x + A_x\left(a_{x1} - \overline{a}_{x1}\right) + B_x\left(b_z - \overline{a}_{z1}\right)\right], \left[1 - \varnothing\left(\alpha_x\left(c, a_{x1}, b_z\right)\right)\right] \\ &+ \left[\overline{\theta}_x + A_x\left(a_{x1} - \overline{a}_{x1}\right) + B_x\left(b_z - \overline{a}_{z1}\right)\right] \varnothing\left(\alpha_x\left(c, a_{x1}, b_z\right)\right) \\ &+ (\gamma - 1)\,\sigma_x^2 \frac{\varnothing\left(\alpha_x\left(c, a_{x1}, b_z\right)\right)}{\sqrt{\vartheta}} - \frac{\beta a_{x1}^2}{2} \end{aligned} \quad (3.8)$$

$$\begin{aligned} U_z\left(a_{z1}, b_x\right) &= w_{z1} + \gamma\left[\overline{\theta}_z + A_z\left(a_{z1} - \overline{a}_{z1}\right) + B_z\left(b_x - \overline{a}_{x1}\right)\right] \varnothing\left(\alpha_z\left(c, a_{z1}, b_x\right)\right)\Big] \\ &+ \left[\overline{\theta}_z + A_z\left(a_{z1} - \overline{a}_{z1}\right) + B_z\left(b_x - \overline{a}_{x1}\right)\right], \left[1 - \varnothing\left(\alpha_z\left(c, a_{z1}, b_x\right)\right)\right] \\ &+ (\gamma - 1)\,\sigma_z^2 \frac{\varnothing\left(\alpha_z\left(c, a_{z1}, b_x\right)\right)}{\sqrt{\vartheta}} - \frac{\beta a_{z1}^2}{2} \end{aligned} \quad (3.9)$$

Where $\varnothing(\bullet)$ and $(\bullet)$ are respectively the cumulative density and probability density function of the standard normal distribution. Worker x chooses a_{x1} to maximizes $U_x(a_{x1}, b_z)$ and z chooses a_{z1} to maximize $U_z\left(a_{z1}, b_x\right)$. Denote by $\left\{a_{x1}^*, a_{z1}^*\right\}$ the Nash equilibrium in this stage, then $b_i = a_{i1}^*$; namely both have perfect foresight about the other's effort. Furthermore, the firm has perfect foresight about both workers' effort as well: $b_i = a_{i1}^* = \bar{a}_{i1}$. Since $\alpha_x\left(c, a_{x1}^*, a_{z1}^*\right) = \alpha_z\left(c, a_{x1}^*, a_{z1}^*\right)$, denote by $\alpha(c)$ the equilibrium value of α_i:

$$\alpha\left(c\right) = \frac{c - E[\,\Delta y|\, a_{x1}^*, a_{z1}^*]}{\sqrt{\vartheta}} = \frac{c - \Delta\bar{\theta} - \Delta a^*}{\sqrt{\vartheta}} \tag{3.10}$$

Where $\Delta a^* = a_{x1}^* - a_{z1}^*$. Since $\varphi(\alpha\left(c\right))$ is the probability that worker z is promoted, and $1-\varphi\left(\alpha\left(c\right)\right)$ is the probability that x is promoted. If $\alpha\left(c\right) < 0$, then worker x has a greater chance to be promoted than z; and vice versa. Given c, the first order condition (The second order condition is satisfied when β is large enough) of maximizing $U_x\left(a_{x1}, b_z\right)$ with respect to a_{x1} in (3.8),evaluated at $b_i = a_{i1}^* = \bar{a}_{i1}$, is

$$\gamma A_x\left[1 - \varphi\left(\alpha\left(c\right)\right)\right] + A_x\varphi\left(\alpha\left(c\right)\right) + \left(\gamma - 1\right)\varphi\left(\alpha\left(c\right)\right)\frac{\bar{\theta}_x}{\sqrt{\vartheta}} + \left(\gamma - 1\right)\varphi\left(\alpha\left(c\right)\right)\alpha\left(c\right)\frac{\sigma_x^2}{\vartheta} = \beta a_{x1}^* \tag{3.11}$$

We say that the rule favors $x\,(z)$ formally if $c < 0$ $(c > 0)$, and that the rule favors $x(z)$ in probability if $\alpha\left(c\right) < 0 (> 0)$. The right-hand side in (3.11) is the marginal cost of effort to worker x, and the left-hand side the incentive to work. There are three kinds of incentives. First is the career concern, including the increase in the expected payoffs from job H (the first term) and job L (the second term) in period 2.

The larger $\alpha(c)$ is, the smaller the probability of x being promoted and therefore the smaller the career concern. When $\alpha\left(c\right)$ goes to infinity, x will be stuck with job L for sure in period 2, and he only cares about the expected wage in job L. In the other extreme, when $\alpha(c)$ goes to negative infinity, x is guaranteed the job H. Since $\gamma > 1$, he works harder the higher the probability that he gets promoted. Lazear and Rosen (1981) argue that it might be optimal to adopt a handicap system to discriminate against the more able worker so that both contestants would have incentive to keep on racing. When workers have career concern, the handicap system might not be necessary.

The second incentive comes from the tournament effect, which is the third term. When $\alpha(c)$ is zero, where $\varphi(\alpha\left(c\right))$ is the largest, the competition is fiercest since each has equal chance of winning job H. As $\alpha(c)$ goes to the two tails, $\varphi(\bullet)$ decreases and so does the competition.

The third effect is in the last term, which we call it the hungry-dog-and-fat-cat effect (We borrow the animal names from Fudenberg & Tirole, 1984.). When $\alpha(c) = 0$, the effect disappear. When $\alpha(c) > 0$, the promotion rule favors z, but worker x exerts more effort instead (hungry dog). And when $\alpha(c) < 0$, x decreases his effort although he is favored (fat cat). Note that this effect is absent in models considering only career concern or tournament. Similarly, the first order condition (evaluated at $b_i = a_{i1}^* = \bar{a}_{i1}$) for worker z's effort a_{z1} includes the above effects too.

$$\gamma A_z \varphi(\alpha(c)) + A_z[1 - \varphi(\alpha(c))] + (\gamma - 1)\varphi(\alpha(c))\frac{\bar{\theta}_z}{\sqrt{\vartheta}} - (\gamma - 1)\varphi(\alpha(c))\alpha(c)\frac{\sigma_z^2}{\vartheta} = \beta a_{z1}^* \tag{3.12}$$

The Nash equilibrium $\{a_{x1}^*, a_{z1}^*\}$ is the solution to the simultaneous equations (3.11) and (3.12).

4. Comparative Statics

To solve the firm's optimal formal promotion rule c, we need to know first how the efforts of workers varies with the rule. Rewrite (3.11) and (3.12) as

$$\begin{aligned} F_x\left(a_{x1}^*, a_{z1}^*\right) &= \gamma A_x[1 - \varphi(\alpha(c))] + A_x \varphi(\alpha(c)) \\ &\quad + (\gamma - 1)\mu(\alpha(c))\frac{\bar{\theta}_x}{\sqrt{\vartheta}} + (\gamma - 1)\mu(\alpha(c))\alpha(c)\frac{\sigma_x^2}{\vartheta} - \beta a_{x1}^* = 0 \end{aligned} \tag{4.1}$$

$$\begin{aligned} F_z\left(a_{x1}^*, a_{z1}^*\right) &= \gamma A_z \varphi(\alpha(c)) + A_z[1 - \varphi(\alpha(c))] \\ &\quad + (\gamma - 1)\mu(\alpha(c))\frac{\bar{\theta}_z}{\sqrt{\vartheta}} - (\gamma - 1)\mu(\alpha(c))\alpha(c)\frac{\sigma_z^2}{\vartheta} - \beta a_{z1}^* = 0 \end{aligned} \tag{4.2}$$

Then we have

$$\frac{\partial a_{x1}^*}{\partial c} = \frac{1}{\cup}\left[\frac{\partial F_x}{\partial a_{z1}}\frac{\partial F_z}{\partial c} - \frac{\partial F_z}{\partial a_{z1}}\frac{\partial F_x}{\partial c}\right] \tag{4.3}$$

$$\frac{\partial a_{z1}^*}{\partial c} = \frac{1}{\cup}\left[\frac{\partial F_z}{\partial a_{x1}}\frac{\partial F_x}{\partial c} - \frac{\partial F_x}{\partial a_{x1}}\frac{\partial F_z}{\partial c}\right] \tag{4.4}$$

where $\cup = \left(\partial F_x / \partial a_{x1}\right)\left(\partial F_z / \partial a_{z1}\right) - \left(\partial F_x / \partial a_{z1}\right)\left(\partial F_z / \partial a_{x1}\right)$ and is assumed to be positive so that the equilibrium is stable. The derivatives of Fi with respect to c are:

$$\frac{\partial F_x}{\partial c} = \frac{-\sigma_x^2(\gamma - 1)\mu(\alpha)}{\vartheta^{1.5}}\left[\alpha^2 + \alpha\frac{\sqrt{\vartheta}\bar{\theta}_x}{\sigma_x^2} + \frac{\vartheta^{1.5}A_x - \sigma_x^2}{\sigma_x^2}\right] \tag{4.5}$$

$$\frac{\partial F_z}{\partial c} = \frac{-\sigma_z^2(\gamma - 1)\mu(\alpha)}{\vartheta^{1.5}}\left[\alpha^2 - \alpha\frac{\sqrt{\vartheta\theta_z}}{\sigma_z^2} + \frac{\vartheta^{1.5}A_z - \sigma_z^2}{\sigma_z^2}\right] \tag{4.6}$$

Let $P_i = \sqrt{\vartheta\theta_i}/\sigma_i^2$, $Q_i = (\vartheta^{1.5}A_i - \sigma_i^2)/\sigma_i^2$, $i = x, z$, then the sign of (4.5) is determined by the sign of the quadratic function $f_x(\alpha) = \alpha^2 + \alpha P_x + Q_x$, and the sign of (4.6) by $f_z(\alpha) = \alpha^2 - \alpha P_z + Q_z$. We say that the mean of worker i's ability is large (small) if $P_i^2 >$ $(<)\, 4Q_i$. When both workers' ability is small, $f_i(\alpha) > 0$ o for all α. When both workers' ability is large, there are two roots of $f_x(\alpha) = 0$ and two of $f_z(\alpha) = 0$, from which we can define the following five sections (note that the first two roots are negative, and the other two positive):

$$S_1 = \left(-\infty, \frac{-P_x - \left(P_x^2 - 4Q_x\right)^{0.5}}{2}\right], \; S_2 = \left[\frac{-P_x - \left(P_x^2 - 4Q_x\right)^{0.5}}{2}, \frac{-P_x + \left(P_x^2 - 4Q_x\right)^{0.5}}{2}\right]$$

$$S_3 = \left[\frac{-P_x + \left(P_x^2 - 4Q_x\right)^{0.5}}{2}, \frac{P_z - \left(P_z^2 - 4Q_z\right)^{0.5}}{2}\right]$$

$$S_4 = \left[\frac{P_z - \left(P_z^2 - 4Q_z\right)^{0.5}}{2}, \frac{P_z + \left(P_z^2 - 4Q_z\right)^{0.5}}{2}\right], S_5 = \left[\frac{P_z + \left(P_z^2 - 4Q_z\right)^{0.5}}{2}, \infty\right] \tag{4.7}$$

We then have the next result.

Proposition 1. Δa^* decreases with c, and $\alpha'(c) > 0$.

Proposition 1 says that the gap of the two workers' effort narrows when the promotion rule favors z more (c larger), and that c has a one-to-one relationship with $\alpha(c)$. But we do not know whether the result is due to that x works harder and z gets lazier, both work harder but z works more, or both work less but x lazier. So we need the next proposition.

Proposition 2. If both workers' mean ability is large, then when $\alpha(c) \in S_2$, $\partial a_{21}^*/\partial c >$ $\partial a_{11}^*/\partial c$, and when α©$\in S_4$, $\partial a_{21}^*/\partial c < \partial a_{11}^*/\partial c < 0$. If both workers' mean ability is small, then $\partial a_{21}^*/\partial c > 0 > \partial a_{11}^*/\partial c$.

Since $\alpha(c)$ increases with c, and $x(z)$ is favored in probability when $\alpha(c) \in S_2(S_4)$, the first two parts of proposition 2 says that when both workers' expected ability is large, and worker i is favored in probability, then increasing further the advantage of the favored would reduce both workers' effort, but k, the unfavored, reduces more than i.

When both workers' expected ability is small, changing the formal rule to favor i would increase $i`s$ effort while reduce $k`s$ effort. Next we examine workers' incentive in the extreme cases to gain more intuition. We say that $x(z)$ is almost sure to be promoted if $\alpha(c) \rightarrow -\infty\,(\infty)$.

5. Promotion Rule and the Firm's Goal

Viewing promotion as the reward to a tournament, Lazear and Rosen (1981) propose that the firm might need to handicap the leader in order to maintain the intensity of the competition, without which neither the leader nor the followers have much incentive to work. However, in a setting where workers' effort is not a concern and the firm only wants to find the best one among his employees, Meyer (1991) shows that the firm should, during the later rounds, give advantage to the leader in the early rounds of tournament. The purpose of the bias is to make the results of the tournament more informative. In this section we will analyze how the firm should design his promotion rule to fit his goal. Both tournament and career concerns affect the incentive of workers to work. In proposition 3, we have shown that a worker still has incentive to work even though he is almost sure to win. T

His extreme case implies that lack of intensity of competition does not reduce the workers' effort as much as in the pure tournament models. On the other hand, favoring an early leader may or may not reduce workers' incentive. When the firm cares more about picking the right person for job H, one might expect that the promotion rule is closer to Meyer's (1991) prescription. Yet we find that it is not the firm's goal that decides the optimal promotion rule.

It is instead the difference between the means and the variance of workers' ability that decides the rule. We first discuss the optimal rule when the firm cares only about picking the best person; that is, the firm wants to maximize his period-2 output. Then we analyze the optimal rule when he wants to maximize his period-1 output.

5.1. Promotion as a screening device

Suppose that the firm cares only about his period-2 output. Since the workers do not exert any effort in period 2, the firm wants to choose the worker with higher ability. His optimal promotion rule is then the value of c that maximizes the probability that the worker who wins the tournament in period 1 has higher ability.

Let the prior distribution of the workers' ability difference be $m(\theta)$, and the conditional probability density function of their output difference be $h\left(\Delta y \mid \Delta\theta\right)$, then

$$m\left(\Delta\theta\right) = \frac{1}{\sqrt{2\pi V(\theta)}}\exp\left[\frac{-(\Delta\theta - \Delta\overline{\theta})^{2}}{2V(\Delta\theta)}\right]$$

$$h\left(\Delta y \mid \Delta\theta\right) = \frac{1}{\sqrt{2\pi V(\theta)}}\exp\left[\frac{-(\Delta y - \Delta a - \Delta\theta)^{2}}{2V(\Delta\varepsilon)}\right] \quad (5.1)$$

The firm chooses c to maximize $T = Pr\left(x\ wins \mid \theta_x > \theta_z\right) + Pr\left(z\ wins \mid \theta_z > \theta_x\right)$, where $Pr(S)$ stands for the probability of the event S:

$$\max_{c}\ T = \int_{0}^{\infty}\int_{c}^{\infty} h\left(\Delta y \mid \Delta\theta\right) d\Delta y m\left(\Delta\theta\right) d\Delta\theta + \int_{-\infty}^{0}\int_{-\infty}^{c} h\left(\Delta y \mid \Delta\theta\right) d\Delta y m\left(\Delta\theta\right) d\Delta\theta \quad (5.2)$$

subject to the constraints that workers' effort is a_{x1}^{*} and a_{z1}^{*}, both of which are functions of c. The first order condition of (5.2) is:

$$\left(1 - \frac{\partial\Delta a^{*}}{\partial c}\right)\left[\int_{-\infty}^{0} h\left(c \mid \Delta\theta\right) m\left(\Delta\theta\right) d\Delta\theta - \int_{0}^{\infty} h\left(c \mid \Delta\theta\right) m\left(\Delta\theta\right) d\Delta\theta\right] = 0 \quad (5.3)$$

Let the posterior density function of $\Delta\theta$ given that $\Delta y =$ be $\hat{m}\left(\Delta\theta \mid \omega\right)$, then we have

$$\hat{m}\left(\Delta\theta \mid \omega\right) = \frac{h\left(\Delta y \mid \Delta\theta\right) m\left(\Delta\theta\right)}{g(\omega)}$$

where $g(\omega)$ is the density function of the normal distribution $N\left(\Delta a^{*}+\Delta\overline{\theta}, \vartheta\right)$, and that $\hat{m}\left(\Delta\theta \mid \omega\right)$ is the density function of the normal distribution $N\left(\mu(\omega), v^{2}\right)$, where

$$\mu\left(\omega\right) = \frac{V\left(\Delta\theta\right)\left(\omega - \Delta a^{*}\right) + V(\Delta\varepsilon)\Delta\overline{\theta}}{\vartheta}$$

And $v^{2} = V\left(\Delta\theta\right) V(\Delta\varepsilon)/\vartheta$, Then (5.3) can be written as

$$\left(1 - \frac{\partial\Delta a}{\partial c}\right)\left[\int_{-\infty}^{0} g(c)\hat{m}\left(\Delta\theta\right) d\Delta\theta - \int_{0}^{\infty} g(c)\hat{m}\left(\Delta\theta\right) d\Delta\theta\right] = 0$$

If we let $Z\left(\Delta\theta \mid c\right) = (\Delta\theta - \mu(c))/v$, where $\hat{m}\left(\Delta\theta\right) = \varphi(Z\left(\Delta\theta \mid c\right)$ is the density function of the standard normal distribution. The first order condition now becomes

$$\left(1 - \frac{\partial\Delta a^{*}}{\partial c}\right) g\left(c\right)\left[2\varnothing\left(\frac{-\mu\left(c\right)}{v}\right) - 1\right] = 0 \quad (5.4)$$

Suppose that $\partial\Delta a^{*}/\partial c \neq 1$, then the firm must set $(c) = 0$ (so that $\varnothing\left(\frac{-\mu(c)}{v}\right) = 0.5$, or

$$c = \Delta a^{*}\left(c\right) - \frac{V\left(\Delta\varepsilon\right)}{V(\Delta\theta)}\Delta\overline{\theta} \quad (5.5)$$

From proposition 1, we know that Δa^* decreases with c. Therefore (5.5) defines the optimal c implicitly, which we denote by c_p. In Figure 1 we draw two possible values of c_p, one is positive and the other is negative. Condition (5.5) says that to maximize the probability that the winner is the more capable worker, the firm's promotion rule is a weighted average of Δa^*, the effort difference of the workers in equilibrium, and $\Delta\overline{\theta}$, the difference of the means of the workers' ability. In an extreme case, if $V(\Delta\varepsilon)/V(\Delta\theta)$ goes to zero, then the variance of Δy should come from Δa^* and so c_p is set to offset the effort difference. On the other hand, if $V(\Delta\varepsilon)/V(\Delta\theta)$ is large, then c_p should give more consideration to $\Delta\overline{\theta}$.

Proposition 3.

1. If $\varphi_x = \varphi_z$ and $= 1$, then $c_p = \Delta a^*$ and $T = 1$.

2. If $V(\Delta\theta) \to \infty$, then $c_p = \Delta a^*$, but $T < 1$.

3. If $V(\Delta\varepsilon)/V(\Delta\theta) \to \infty$, then $c_p \to \infty$ if $\Delta\overline{\theta} > 0$, and $c_p \to -\infty$ if $\Delta\overline{\theta} < 0$.

4. If $\Delta\overline{\theta} = 0$, $\varphi_x = \varphi_z$, $\rho = 1$, and $\sigma_x = \sigma_z$, then $c_p = 0$.

5. $\alpha(c_p)$ is positive if $\Delta\overline{\theta}$ is negative, and vice versa.

Proof. Parts (1) to (3) follow directly from (5.5). When the condition in part (4) holds, $a^*_{x1} = a^*_{z1}$ and hence $c_p = \Delta a^* = 0$. To show part (5), we insert c_p into (3.10) and get

$$\alpha(c_p) = \frac{-\Delta\overline{\theta}}{\sqrt{\vartheta}}\left[1 + \frac{V(\Delta\varepsilon)}{V(\Delta\theta)}\right]$$

Part (5) of proposition 4 can complement Meyer's (1991) promotion rule. In a setting where workers' outputs are independent of effort, Meyer shows that to maximize the probability that the firm promotes the most capable worker, the firm should give edge to leader of early rounds of competition over the followers; that is, setting $c < 0$ if x is the leader. In our setting, the sign of c_p has no fixed relationship with the sign of $\Delta\overline{\theta}$, while the sign of $\alpha(c_p)$ is the opposite of $\Delta\overline{\theta}$. Hence, the early leader (the one whose expected ability is larger) enjoys the edge in probability term instead of nominal term.

5.2. Promotion as an incentive device

Suppose that the firm does not care about his period-2 output, and uses promotion solely to maximize period-1 outputs. Since the period-1 wages are independent of outputs, the firm's problem is

$$\max_{C}\; a_{x1}^{*}(c) + a_{z1}^{*}(c)$$

where $a_{x1}^{*}(c)$ and $a_{z1}^{*}(c)$ are the solution to (3.11) and (3.12). From (4.1) and (4.2), we have

$$\begin{aligned}\beta\left(a_{x1}^{*}(c) + a_{z1}^{*}(c)\right) \equiv {} & (\gamma-1)\left(A_z - A_x\right)\varnothing(\alpha(c)) + \gamma A_x + A_z \\ & + \frac{(\gamma-1)\,\mu(\alpha(c))}{\sqrt{\vartheta}}(\overline{\theta}_x + \overline{\theta}_z) + \frac{(\gamma-1)\,\mu(\alpha(c))\,\alpha(c)}{\vartheta}(\sigma_x^2 - \sigma_z^2)\end{aligned} \tag{5.6}$$

Let $L(\alpha(c))$ denote the right hand side of (5.6). Since c affects $L(\alpha(c))$ only through $\alpha(c)$, the firm's objective becomes maximizing $L(\alpha(c))$ by $\alpha(c)$. The first order condition is

$$\begin{aligned}L`(\alpha(c)) = \frac{-(\gamma-1)\,\mu(\alpha(c))}{\vartheta}\; & [(\sigma_x^2 - \sigma_z^2)(\alpha(c))^2 + \sqrt{\vartheta}\left(\overline{\theta}_x + \overline{\theta}_z\right)\alpha(c) \\ & + \vartheta\left(A_z - A_x\right) - (\sigma_x^2 - \sigma_z^2)] = 0\end{aligned} \tag{5.7}$$

and the second derivative of L is

$$\begin{aligned}L``(\alpha(c)) = \frac{(\gamma-1)\,\mu(\alpha(c))}{\vartheta}\; & [(\sigma_x^2 - \sigma_z^2)(\alpha(c)^3 - 3\alpha(c)) + \sqrt{\vartheta}\left(\overline{\theta}_x + \overline{\theta}_z\right)(\alpha(c)^2 - 1) \\ & + \vartheta\left(A_z - A_x\right)\alpha(c)]\end{aligned}$$

Denote the terms in the square brackets of (5.7) by the function l$(\alpha(c))$, then $L` = 0$ if and only if $l = 0$. Let the solution be $\alpha(c_\pi)$, then the second order condition requires that

$$L``(\alpha(c)) = -2\alpha(c_\pi)(\sigma_x^2 - \sigma_z^2) - \sqrt{\vartheta}\left(\overline{\theta}_x + \overline{\theta}_z\right) < 0 \tag{5.8}$$

Since l$(\alpha(c))$ is a quadratic function of $\alpha(c)$, $l = 0$ has roots if and only if

$$R \equiv \vartheta\left(\overline{\theta}_x + \overline{\theta}_z\right)^2 - 4(\sigma_x^2 - \sigma_z^2)[\vartheta\left(A_z - A_x\right) - (\sigma_x^2 - \sigma_z^2)] \geq 0$$

Suppose that $R \geq 0$, then the solution to l$(\alpha(c))$ that satisfies (5.8) is

$$\alpha\left(c_{\pi}\right)=\frac{1}{2}\left[\frac{\sqrt{R}-\sqrt{\vartheta}\left(\bar{\theta}_{x}+\bar{\theta}_{z}\right)}{\sigma_{x}^{2}-\sigma_{z}^{2}}\right] \tag{5.9}$$

We summarize a few special cases in the next proposition.

Proposition 4. Suppose that $R \geq 0$.

1. If $\varphi_x^2 = \varphi_z{}^2$ *and* $\sigma_x^2 > (respcetivly <)\ \sigma_z^2$, then $\alpha\left(c_\pi\right) > (respcetivly <)\ 0$
2. If $\sigma_x^2 = \sigma_z^2$, then $\alpha(c_\pi) = \sqrt{\vartheta}\left(A_x - A_z\right)/\bar{\theta}_x + \bar{\theta}_z$, which is positive (negative) if $\varphi_z^2 > (<) \varphi_x^2$.
3. If $\sigma_x^2 = \sigma_z^2$ and $\varphi_x^2 = \varphi_z{}^2$, then $\alpha(c_\pi) = 0$.
4. If $\sigma_x^2 = \sigma_z^2$, $\varphi_x^2 = \varphi_z^2 ,= 1$, and $\bar{\theta}_x = \bar{\theta}_z$, then$\alpha\left(c_\pi\right) = c_\pi = 0$.

Proof. If $\varphi_x^2 = \varphi_z{}^2$, then from the definition of A_i in (3.2), we have

$$\sigma_x^2 > (<,\ =)\,\sigma_z^2 \leftrightarrow A_z > (<,=) A_x$$

which proves part (1). Part (2) follows from (5.7). The condition in part (3) implies that $A_x = A_z$, and hence by part (2), $\alpha(c_\pi) = 0$. Part (4) follows from part (3) and (3.10).

5.3. Comparisons

We have shown the optimal promotion rules for the two different goals of the firm. Here we shall compare how the two rules, c_p and c_π, differ. We start with some special cases. From part (5) of proposition 4, we know that to maximize the probability that the best worker is promoted, the promotion rule favors the leader (the one with higher expected ability) in probability. But from proposition 5, we see that when the firm wants to maximize period-1 outputs, the optimal promotion probability does not depend on $\Delta\bar{\theta}$. This illustrates why there is a conflict for a promotion rule to serve both incentive and screening purposes. However, the two optimal rules might favor the same worker. The next corollary summarizes these cases.

Corollary 1. $\alpha(c_p)$ and $\alpha(c_\pi)$ have the same signs in the following cases:

1. If $\Delta\bar{\theta} = 0$, $\sigma_x = \sigma_z$, and $\varphi_x = \varphi_z$, then $\alpha\left(c_p\right) = \alpha\left(c_\pi\right) = 0$.

2. If $\Delta\bar{\theta} > 0\,(respectively < 0)$, $\varphi_x = \varphi_z$, and $\sigma_x < (respectively >)\ \sigma_z$, then both $\alpha\left(c_p\right)$ and $\alpha\left(c_\pi\right)$ are negative (respectively, positive).

3. If $\Delta\bar{\theta} > 0\,(respectively < 0)$, $\sigma_x = \sigma_z$, and $\varphi_x > (respectively < 0)$, then both$\alpha\left(c_p\right)$ and $\alpha\left(c_\pi\right)$ are negative (respectively, positive).

6. Conclusion

We have developed a model of promotion that encompasses both career concerns and tournament effect. We first shows that workers' effort incentive is not only affected by both career concerns and tournament effect, but also a combinative effect. Then we analyze how the firm's promotion rules should vary with his goals and the parameters.

When the firm wants to pick the best worker, the promotion rule favors in probability the worker with higher expected ability. When the firm wants to maximize his period-1 outputs, the promotion rule might favor either worker, depending on the variances of their ability and the disturbance. Since the latter rule does not depend on the workers' expected difference of ability, the two rules can differ. However, under some conditions, the two rules are consistent; namely the two rules both favor the same worker.

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